

SECOND ORDER ANALYSIS EXAMPLE

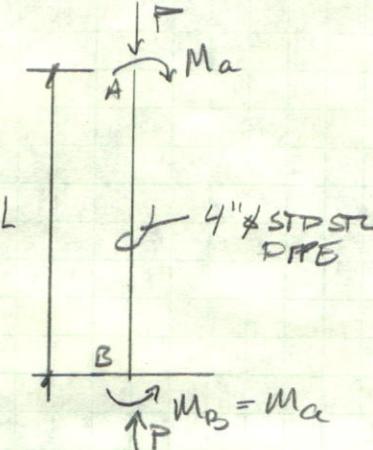
$$P = 500 \text{ #} \quad I = 6.82 \text{ in}^4$$

$$M_A = 1' \text{ K}$$

$$L = 15'$$

ON UNDEFLECTED SHAPE:

$$M_B = M_A$$



LOADS ON UNDEFLECTED SHAPE

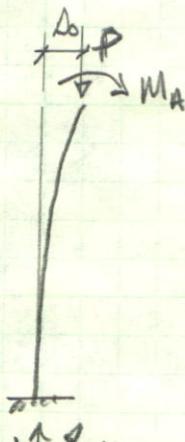
DEFLECTION @ A DUE TO MOMENT, Ma

$$\Delta_0 = \frac{M_A L^2}{2EI} = \frac{1000' \# (15')^2 (1728 \text{ in}^3 / 12)}{2(29 \times 10^6 \#/\text{in}^2)(6.82 \text{ in}^4)} = .983"$$

RECOMPUTE Mb

$$\begin{aligned} M_B &= M_A + P\Delta_0 \\ &= 1000' \# + 500' \# \left( \frac{.983}{12} \right) \end{aligned}$$

$$M_B = 1041' \#$$



THIS CAUSES ADDITIONAL DEFL:

$$\Delta_T = \Delta_0 + \Delta_1 = \frac{(M_0 + M_1)L^2}{2EI} \quad \Delta_0 = .983" \quad \Delta_1 = .0403" \quad \Delta_T = 1.023"$$

$$\Delta_1 = .983" + \frac{41.0' \# (15')^2 (1728)}{2(29,000,000)(6.82)}$$

$$M_B = M_A + P\Delta_0$$

$$\Delta_T = 1.023"$$

RECOMPUTE Mb

$$M_B = M_0 + M_1 + P\Delta_1 \quad 1.68"$$

$$M_B = 1041' \# + 500' \# (.0403")$$

RECOMPUTE Δ

RECOMPUTE Mb

UNTIL  $\Delta_i \neq M_i$  ARE VERY SMALL

SAME PROBLEM USING A TABLE:

STAGE	STAGE MOMENT	M <sub>B</sub>	STAGE DEFL	ΔA
	M <sub>i</sub> (FT-#)	(PT-#)	Δ <sub>i</sub> (in)	(in)
0	1000	1000	.983	0.983
1	41.0	1041.0	.0403	1.023
2	1.68	1042.6	1.65 × 10 <sup>-3</sup>	1.025
3	68.7 × 10 <sup>-3</sup>	1042.9	67.5 × 10 <sup>-6</sup>	1.025

CHANGES ARE INSIGNIFICANT AFTER  
2<sup>nd</sup> STAGE ∴ CONVERGED

INCREASE IN MOMENT DUE TO 2<sup>nd</sup> ORDER EFFECTS:

$$\frac{1044.3 - 1000}{1000} = \underline{\underline{4.43\%}}$$